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HEAT TRANSFER TO VARIABLE PROPERTY FLUIDS IN TURBULENT PIPE FLOW: A TRANSFORMATION FOR A PARTICULAR CASE OF PROPERTY VARIATIONS

M. B. IBRAHIM and V. WALKER
University of Bradford, Bradford, West Yorkshire BD7 1DP, England

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NOMENCLATURE

c_p, specific heat at constant pressure;
d, pipe diameter;
f, friction factor, $\dfrac{\tau_w}{\frac{1}{2}\rho u_b^2}$;
k, thermal conductivity;
$\dot{q}$, heat flux per unit area;
t, temperature;
t^+, dimensionless temperature, $\dfrac{(t_w - t)c_{pw}\tau_w}{\dot{q}_w u^*}$;
T, absolute temperature;
u, velocity;
u^*, friction velocity, $\left(\dfrac{\tau_w}{\rho}\right)^{1/2}$;
x, axial coordinate;
X, x/d;
y, distance from pipe surface;
y^+, u^*y/v;
$\bar{c}_{pwb}$, average c_p over temperature range t_w to t_b;
$\bar{k}_{wl}$, average k over temperature range t_w to t_l;
$\bar{c}_{plb}$, average c_p over temperature range t_l to t_b.

Greek symbols

ρ, density;
μ, viscosity;
v, μ/ρ;
τ_w, wall shear stress;
ε, eddy diffusivity of momentum;
ε_h, eddy diffusivity of heat;
λ, fraction of temperature difference occurring across sub-layer;
Nu, Nusselt number;
Pr, Prandtl number;
Pr_t, turbulent Prandtl number, $\varepsilon/\varepsilon_h$;
Re, Reynolds number;
St, Stanton number.

Suffixes

b, bulk;
l, at edge of sub-layer;
r, reference;
w, wall;
0, constant property value.

INTRODUCTION

THE SCOPE of this paper is heat transfer to (or from) fluids in turbulent flow through smooth pipes with wall–fluid temperature differences large enough to produce significant variations of the relevant physical properties c_p, k, ρ, μ, but not sufficiently large to require consideration of buoyancy or acceleration effects; compressibility also is excluded. This situation is of great engineering importance as evidenced by the large number of experimental and theoretical investigations and reviews devoted to it.

The relevant dimensionless variables are well known, i.e.

$$Nu_b = f(Re_b, Pr_b, c_{pw}/c_{pb}, k_w/k_b, \rho_w/\rho_b, \mu_w/\mu_b)$$

provided also that the forms of the variations of the four physical properties over the relevant temperature range are specified. Development of a comprehensive correlation involving so many variables is a formidable task. Experimentally it requires a large body of reliable data. Extension of the semi-theoretical methods which have been used for the constant-property case can only be tentative, particularly where ρ and μ variations are involved owing to the coupling of the momentum and energy equations. Inevitably the many correlations proposed so far have involved simplifying assumptions either about the form of the above mathematical function or the adequacy of reference values of the physical properties. In practice, use of alternative correlations for design purposes frequently reveals discrepancies which are unacceptable.

For heat conduction in solids there exists a mathematical transformation which enables solutions for temperature-dependent thermal conductivity to be deduced from known solutions for constant conductivity. In this paper the transformation is extended to heat transfer in pipe flow with coupled c_p and k variations, and its usefulness in the search for a well-based general correlation is then examined.

TURBULENT PIPE FLOW WITH VARIABLE SPECIFIC HEAT AND THERMAL CONDUCTIVITY

We make the usual assumption that the effective conductivity can be expressed as the sum of a fluid and a turbulent conductivity:

$$\dot{q} = -(k + \rho c_p \varepsilon_h)\frac{dt}{dy}. \tag{1}$$

We now consider (hypothetical) fluids for which density and viscosity are constant, while k and c_p vary identically with temperature, i.e.

$$\frac{k}{k_b} = \frac{c_p}{c_{pb}}.$$

Hence Pr = constant = Pr_b.

Furthermore, since the turbulent Prandtl number, $Pr_t = F(Re, Pr, y^+)$, it is not affected by the coupled variation of c_p and k. We need no further information about Pr_t or ε/v in the following analysis and therefore assume no particular turbulence model.

In dimensionless form equation (1) becomes

$$\frac{\dot{q}}{\dot{q}_w} = \frac{c_p}{c_{pw}}\left(\frac{1}{Pr} + \frac{1}{Pr_t}\frac{\varepsilon}{v}\right)\frac{dt^+}{dy^+} \tag{2}$$

where the factor within brackets is unaffected by the variation of c_p and k. We now consider the corresponding equation for the temperature t_0^+ in the constant property case, take the radial variation of heat flux, $\dot{q}/\dot{q}_w$, to be the same in both cases (this is discussed in the Appendix), and hence obtain

$$\frac{c_p}{c_{pw}}\frac{dt^+}{dy^+} = \frac{dt_0^+}{dy^+} \quad \text{for equal } Re \text{ and } Pr.$$

Hence

$$\frac{1}{c_{pw}}\int_0^{t^+} c_p\,\mathrm{d}t^+ = \frac{1}{k_w}\int_0^{t^+} k\,\mathrm{d}t^+ = t_0^+. \tag{3}$$

This is recognisable immediately as the exact analogy of the temperature transformation in heat-conduction problems with variable conductivity. The distribution of the transformed temperature, given by either of the integrals, is identical to the distribution t_0^+ in the constant property case (for the same Re and Pr).

It also follows from (3) that

$$\frac{1}{c_{pw}}\int_0^{t_b^+} c_p\,\mathrm{d}t^+ = \frac{t_b^+\,\bar{c}_{p,wb}}{c_{pw}}$$

is constant for fixed values of Re and Pr. Therefore

$$\frac{\bar{c}_{p,wb}(t_w - t_b)\tau_w}{\dot{q}_w u^*} = \phi(Re, Pr)$$

and

$$\frac{\dot{q}_w}{(t_w - t_b)\rho u_t \bar{c}_{p,wb}} = \frac{(f/2)^{1/2}}{\phi(Re, Pr)} = \psi(Re, Pr).$$

Thus for the special cases of coupled c_p and k variations (of any form), all heat-transfer results are reduced to the form of the constant property correlation, $St = \psi(Re, Pr)$, by forming St and Pr from the mean values of c_p and k over the temperature range t_b to t_w. The mean c_p has been used previously, particularly in heat-transfer correlations for supercritical pressure fluids where it appeared essential to include weighting for the very wide range of c_p, including localised peaks. It is perhaps surprising that a mean conductivity does not seem to have been considered in view of its established use in conduction problems.

Before we proceed to examine the significance of the above transformation it is worth commenting upon the somewhat surprising fact that specific results are derived without an eddy diffusivity model. Firstly, the eddy diffusivity approach postulates two heat-transfer processes in parallel, a laminar and a turbulent conductivity. The coupling of c_p and k, and the exclusion of viscosity and density variations, are sufficient to ensure that, at any radial position, both heat-transfer processes are affected to the same extent while the flow structure is unaffected. Secondly, the results relate only to St/St_0; absolute values of St would require a detailed model.

THE SIGNIFICANCE OF THE TRANSFORMATION

What is the value of results, even fairly rigorous results, for a hypothetical fluid for which only c_p and k are temperature dependent and coupled such that c_p/k is constant? Perhaps as a partial test of the soundness of theoretical calculations or empirical correlations. However, in the case of theoretical calculations, no test is provided for the correctness of the eddy diffusivity model since we have noted already that any such model leads to the present results for coupled c_p and k. These results just provide a check on the computational soundness and internal consistency of the calculations for variable physical properties.

The present results are likely to be of most use as a test of the many existing correlations embracing variable properties or in providing guidance for the development of new ones. As noted in the Introduction, a comprehensive correlation poses a formidable problem, most workers have resorted to drastic simplifying assumptions, and the designer is faced with a confusion of correlations and a wide spread of design data for a particular application. In some instances the irregularities in a correlation may be obvious on inspection [1]. Comparison with the present results for c_p and k variations may be a useful further check. The objection might be raised that it would not be reasonable to expect a practical correlation to cover a hypothetical fluid. In reply one might point out, firstly, that although a fluid with only c_p and k variations, directly coupled, is a hypothetical one it is not theoretically unacceptable; secondly, that a test involving only c_p and k variations is not excessively onerous. We now need to look briefly at typical forms of correlation.

Correlations containing property ratios

Although often considered in principle, actual correlations of this type are rare, except for liquids where only the viscosity variation, μ_w/μ_b, is considered significant. For gases, all four properties may be temperature dependent but, on the basis that each depends upon the absolute temperature, the property ratios are usually lumped together and their combined effect represented by a function (which varies with the gas as well as with the author) of T_w/T_b. These are not general correlations and it is not possible to separate the effects of c_p and k for comparison with the present work. Similarly, many correlations for near-critical fluids are purely empirical, with factors based on just one or two properties representing the effects of all four. One of the more explicit correlations has been given by Petukhov *et al.* [2]:

$$Nu_b = Nu_0\left(\frac{\mu_w}{\mu_b}\right)^{0.11}\left(\frac{k_w}{k_b}\right)^{0.33}\left(\frac{\bar{c}_{p,wb}}{c_{pb}}\right)^{0.35}$$

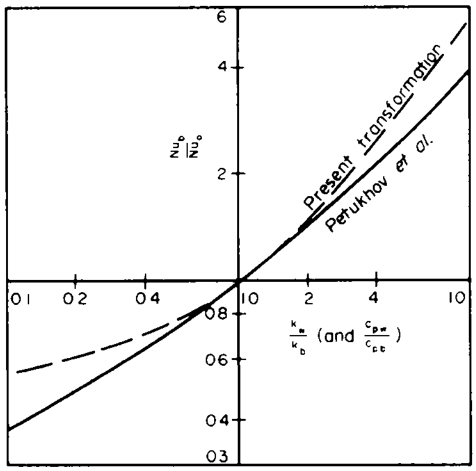

FIG. 1.

For the case of coupled c_p and k variations (with ρ and μ constant), and in particular c_p and k varying linearly with temperature, Petukhov's equation becomes

$$Nu_b = Nu_0\left(\frac{k_w}{k_b}\right)^{0.33}\left[\frac{1}{2}\left(1+\frac{k_w}{k_b}\right)\right]^{0.35}$$

whereas the transformation described above yields

$$Nu_b = \frac{Nu_0}{2}\left(1+\frac{k_w}{k_b}\right).$$

The two forms are compared in Fig. 1. It is seen that the agreement is good for $0.25 < k_w/k_b < 4.0$. For lower values of k_w/k_b the factor $(k_w/k_b)^{0.33}$ in Petukhov's correlation overstresses the effect of k_w. This limited test by no means validates Petukhov's correlation, but it helps to define the range over which it might be a fair approximation. The correlation might have been improved by representing the conductivity variation by $(\bar{k}_{wb}/k_b)$ to a power of about 0.65.

Correlations involving fluid properties at a reference temperature

It has often been assumed that correlations established for constant properties could be continued unchanged for variable property situations provided one could identify the correct reference temperature at which to evaluate the physical properties. This method is open to question, for example it is very doubtful whether the correct weighting can be given to all four properties by evaluating them at the same temperature. Leaving that aside, we recall that for coupled c_p and k variations, heat-transfer results are completely correlated by introducing $\bar{c}_{p_{wb}}$ and $\bar{k}_{wb}$. If k and c_p vary linearly with temperature,

$$\bar{k}_{wb} = \frac{k_w + k_b}{2} = k_r$$

provided

$$t_r = \frac{t_w + t_b}{2}.$$

Therefore, for property variations small enough to be regarded as linear, the conventional film temperature satisfies the present limited test. No single choice of reference temperature could be correct for non-linear property variations.

The rational choice of reference values for k *and* c_p

In the preceding illustrative examples we have been testing correlations which, although arbitrary in form, might at least prove to be acceptable approximations. However our real objective is the development of more satisfactory forms of correlation, and we enquire whether $\bar{k}_{wb}$ and $\bar{c}_{p_{wb}}$ might have an extended usefulness beyond those special cases in which the temperature dependence of k and c_p are identical. Unfortunately there is no good reason to expect such an outcome. We recall that the transformation presented in this paper is valid because, with coupled c_p and k, the laminar and turbulent conductivities are affected to precisely the same extent. The emergence of $\bar{k}_{wb}$ and $\bar{c}_{p_{wb}}$ does not signify that either k or c_p is significant over the full temperature range t_w to t_b, but that k and c_p *together* control the heat-transfer process over the full temperature range. We are at present engaged upon extensive calculations of the effect of physical property variations, using a particular eddy diffusivity model, and have confirmed that a correlation based upon $\bar{k}_{wb}$ and $\bar{c}_{p_{wb}}$ fails in situations where k and c_p do not vary identically. This is hardly surprising.

While the use of average properties seems inherently promising, if we are to make further progress we might expect to have to determine appropriate averages of k and c_p reflecting the regions of the pipe cross-section in which each is important. For example, if there was a truly laminar sub-layer it would not seem appropriate to include that part of the temperature range in forming an average c_p; conversely, an average k might be based upon just that laminar sub-layer part of the temperature range. A fairly straightforward development confirms this expectation and leads to the conclusion that a constant property correlation could be extended to embrace variations of c_p and k, not necessarily coupled, through the introduction of

$$\bar{k}_{wl}, \text{ the average over } 0 < t^+ < \lambda t_b^+$$

and

$$\bar{c}_{plb}, \text{ the average over } \lambda t_b^+ < t^+ < t_b^+$$

where λ is the fraction of the temperature difference occurring across the sub-layer in the constant property case. A simple laminar/turbulent two-layer model can be a tolerable approximation for Pr not too far from unity, which includes all gases. Hence this development is potentially useful.

It is possible to deduce an effective value for λ. The steps in the argument are as follows:

(a) For Pr not too far from unity, the constant property data can be represented by $Nu_0 = C\,Re_0^m Pr_0^n$.

(b) This correlation can be extended to variable k and c_p by introducing $\bar{k}_{wl}$ and $\bar{c}_{plb}$.

(c) The correlation (a) can also be extended to the special case of coupled k and c_p variations by introducing $\bar{k}_{wb}$ and $\bar{c}_{p_{wb}}$. For this special case, the alternative forms (b) and (c) must be equivalent. This leads to the conclusion that, for relatively small property variations, $\lambda = 1-n$, say 0.6.

In conclusion, we note again that it is questionable whether a completely general correlation is attainable, even when limited to moderate property variations for which such additional effects as buoyancy are not significant. A series of correlations, each with its defined range of application, seems a more realistic objective. In devising such correlations to represent the results of both experiment and computation it is desirable to proceed on a more rational basis than can be discerned in the existing arbitrary forms. It is our hope that the transformation described in this paper provides hints for the construction of more useful correlations.

The transformation, like the corresponding one for conduction problems, is applicable, in principle, to a variety of configurations, not just smooth pipes. For each case it would be necessary to justify that the distribution $\dot{q}/\dot{q}_w$ is not significantly altered by the property variations.

REFERENCES

1. M. B. Ibrahim and V. Walker, Correlations for heat transfer to variable property fluids in turbulent pipe flow, *Int. J. Heat Mass Transfer* **19**, 126 (1976).
2. B. S. Petukhov, E. A. Krasnoschekov and V. S. Protopopov, An investigation of heat transfer to fluids flowing in pipes under supercritical conditions, *Proc. 1961 Int. Heat Transfer Conference, Boulder, U.S.A.*, pp. 569–578 (1961).

APPENDIX

The Radial Variation of Heat Flux, $\dot{q}/\dot{q}_w$

In a discussion of a fully-developed heat-transfer situation it is necessary to specify the thermal boundary condition along the pipe. We start from the basic case of uniform wall heat flux. For the constant property situation, the fully-developed state includes a radial temperature profile of unchanging form and $\partial t/\partial x$ independent of radial position. For the variable property case the properties are varying in the x-direction owing to the temperature gradient and the existence of a fully-developed situation is not self-evident. It is therefore unavoidable that we compare the fully-developed constant property case with a less well defined, not strictly fully-developed, variable property situation.

There are reasons for the belief that this complication will not interfere seriously with the comparison:

(a) Physical property gradients in the axial direction are much smaller than in the radial direction.

(b) Quite drastic approximations to the true $\dot{q}/\dot{q}_w$ distribution are known to produce only minor changes in calculated Nusselt numbers.

We have also examined the problem from another point of view: What axial variation of $\dot{q}_w$ would produce the same radial variation $\dot{q}/\dot{q}_w$ as in the constant property case? Approximate calculations then lead to $(1/\dot{q}_w)(d\dot{q}_w/dX) = 4St$ which is small compared with unity for a wide range of Re and Pr. We are therefore comparing

(a) the fully-developed Nu_0 for uniform heat flux with

(b) Nu (not truly fully-developed) for a slowly varying wall heat flux.

The fact that Nu is known to be insensitive to axial variations of $\dot{q}_w$ reinforces the belief that this is a satisfactory basis for examining the effect of physical property variations.